

All entangled states can demonstrate non-classical teleportation

Daniel Cavalcanti,¹ Paul Skrzypczyk,² and Ivan Šupić¹

¹ICFO-Institut de Ciències Fòniques, The Barcelona Institute of Science and Technology, 08860 Castelldefels (Barcelona), Spain*

²H. H. Wills Physics Laboratory, University of Bristol, Tyndall Avenue, Bristol, BS8 1TL, United Kingdom

Quantum teleportation, the process by which Alice can transfer an unknown quantum state to Bob by using pre-shared entanglement and classical communication, is one of the cornerstones of quantum information. The standard benchmark for certifying quantum teleportation consists in surpassing the maximum average fidelity between the teleported and the target states that can be achieved classically. According to this figure of merit, not all entangled states are useful for teleportation. Here we propose a new benchmark that uses the full information available in a teleportation experiment and prove that all entangled states can implement a quantum channel which can not be reproduced classically. We introduce the idea of non-classical teleportation witness to certify if a teleportation experiment is genuinely quantum and discuss how to quantify this phenomenon. Our work provides new techniques for studying teleportation that can be immediately applied to certify the quality of quantum technologies.

Quantum teleportation [1] is one the cornerstones of quantum information science, and serves as a primitive in several quantum information tasks [2–4]. After the first demonstrations [5–7], quantum teleportation has been demonstrated in a variety of physical systems and become a tesbed for quantum information platforms [8]. In the ideal case, quantum teleportation refers to the situation where Alice and Bob share a maximally entangled state that they can use to transmit quantum information. A verifier, who wants to check that Alice and Bob are making use of entanglement in the teleportation process, provides quantum systems to Alice in states $|\psi_x\rangle$ which are unknown to her, and asks her to transmit these states to Bob. By applying a Bell-state measurement on the input system and her share of the maximally entangled state, Alice projects Bob's system into the states $\rho_a^B|\psi_x\rangle = U_a|\psi_x\rangle\langle\psi_x|U_a^\dagger$, where U_a is a known unitary operation that depends on the outcome a of the Bell state measurement. Bob then sends his system to the verifier, who checks whether the states that Bob sent are the same as the ones he provided to Alice, modulo the conditional unitary transformation.

In any realistic implementation, the state shared by Alice and Bob is not maximally entangled, and the measurement Alice applies is not a perfect Bell state measurement. In this case the states that Bob receives after Alice applies a measurement M with measurement operators M_a^{VA} on systems V and A are given by:

$$\rho_a^B|\psi_x\rangle = \frac{\text{tr}_{VA}[(M_a^{VA} \otimes \mathbb{I}^B) \cdot (|\psi_x\rangle\langle\psi_x|^V \otimes \rho^{AB})]}{p(a|\psi_x)}, \quad (1)$$

where ρ^{AB} is the state shared by Alice and Bob, and $p(a|\psi_x) = \text{tr}[(M_a^{VA} \otimes \mathbb{I}^B) \cdot (|\psi_x\rangle\langle\psi_x|^V \otimes \rho^{AB})]$ is the probability of the particular outcome a given that the verifier gives to Alice the state $|\psi_x\rangle$. The aim of the verifier is then to certify, based on the knowledge of $\{|\psi_x\rangle\}_x$, $\{\rho_a^B|\psi_x\rangle\}_{a,x}$ and $\{p(a|\psi_x)\}_{a,x}$, that Alice and Bob have used a quantum channel, *i.e.* that ρ^{AB} is entangled. Only if he concludes that this is the case will he be convinced that Alice and Bob have performed non-classical teleportation.

The figure-of-merit used to certify quantum teleportation is the average fidelity between the input and output states of the

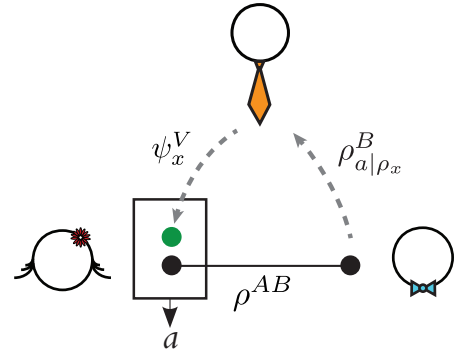


Figure 1. Teleportation scenario: Alice and Bob share a bipartite state ρ^{AB} . A verifier, who wants to check whether this state is entangled, sends systems in one of the states $|\psi_x^V\rangle$ to Alice, and asks her to transmit it to Bob. Alice applies a global measurement on the state given to her by the verifier and her share of ρ^{AB} , which produces the states $\rho_a^B|\psi_x\rangle$ for Bob. The verifier has to determine if ρ^{AB} is entangled based on the knowledge of $\{|\psi_x^V\rangle\}_x$ and $\{\rho_a^B|\psi_x\rangle\}_{a,x}$.

process [9]:

$$\overline{F}_{\text{tel}} = \frac{1}{|x|} \sum_{a,x} p(a|\psi_x) \langle \psi_x | U_a \rho_a^B |\psi_x\rangle U_a^\dagger | \psi_x \rangle. \quad (2)$$

Clearly, in the case of perfect teleportation, $\overline{F}_{\text{tel}} = 1$, while in real experiments one always obtains smaller values. If the state shared by Alice and Bob is separable, the maximum fidelity of teleportation that they can obtain, called the classical average fidelity, is denoted by \overline{F}_{cl} . Quantum teleportation is certified if $\overline{F}_{\text{tel}} > \overline{F}_{\text{cl}}$ [8].

It turns out that some entangled states can not lead to $\overline{F}_{\text{tel}} > \overline{F}_{\text{cl}}$ (*e.g.* bound entangled states) [10]. Thus, according to this benchmark, these entangled states are useless for teleportation (although they can help in improving $\overline{F}_{\text{tel}}$ of a combined state [11]). However, notice that one has more information in a teleportation experiment than simply the value of $\overline{F}_{\text{tel}}$. Thus, in principle, there could exist a situation in which $\overline{F}_{\text{tel}} \leq \overline{F}_{\text{cl}}$, but for which no classical teleportation scheme could explain the full data observed in the experiment. Here

we show that this is indeed the case. We propose a method to certify the quantumness of a teleportation experiment that uses the full information available and use it to prove that all entangled states can be used to implement teleportation channels that are genuinely quantum. Furthermore this is true even with incomplete Bell state measurements, or when utilising inefficient detectors.

For convenience, in what follows we will work with the set of unnormalised teleported states

$$\begin{aligned}\sigma_{a|\psi_x}^B &= \text{tr}_{VA}[(M_a^{VA} \otimes \mathbb{1}^B) \cdot (\psi_x^V \otimes \rho^{AB})] \\ &= \text{tr}_V[M_a^{VB}(\psi_x^V \otimes \mathbb{1}^B)],\end{aligned}\quad (3)$$

where the state given to Alice by the verifier is now simply denoted ψ_x^V and does not need to be a pure state, and

$$M_a^{VB} = \text{tr}_A[(M_a^{VA} \otimes \mathbb{1}^B) \cdot (\mathbb{1}^V \otimes \rho^{AB})]. \quad (4)$$

The normalisation factor $p(a|\psi_x) = \text{tr}[\sigma_{a|\psi_x}^B]$ is nothing but the probability that Alice receives outcomes a given that the input state was ψ_x^V . Equation (3) describes teleportation as a collection of channels from V to B, labelled by a , that transforms the input states ψ_x^V into the (unnormalised) output states $\sigma_{a|\psi_x}^B$, according to the channel operators M_a^{VB} . Notice that in the case that the set $\{\psi_x^V\}_x$ is tomographically complete, this becomes a typical instance of quantum process tomography [9], and the operators M_a^{VB} can be explicitly reconstructed by the verifier.

Suppose now that ρ^{AB} is a separable state, *i.e.* $\rho^{AB} = \sum_\lambda p_\lambda \rho_\lambda^A \otimes \rho_\lambda^B$, such that Alice and Bob are only carrying out classical teleportation. In this case the channel operators (4) become:

$$\begin{aligned}M_a^{VB} &= \sum_\lambda p_\lambda \text{tr}_A[(M_a^{VA} \otimes \mathbb{1}^B) \cdot (\mathbb{1}^V \otimes \rho_\lambda^A \otimes \rho_\lambda^B)] \\ &= \sum_\lambda p_\lambda M_{a|\lambda}^V \otimes \rho_\lambda^B,\end{aligned}\quad (5)$$

where

$$M_{a|\lambda}^V = \text{tr}_A[M_a^{VA}(\mathbb{1}^V \otimes \rho_\lambda^A)]. \quad (6)$$

In this case Eq. (3) becomes

$$\sigma_{a|\psi_x}^B = \sum_\lambda p_\lambda \text{tr}[M_{a|\lambda}^V \psi_x^V] \rho_\lambda^B. \quad (7)$$

This can be understood as describing a purely classical teleportation channel: a classical variable λ is sampled from p_λ and sent to Alice and Bob. Upon receiving λ Alice measures the verifiers system V using the measurement operators $\{M_{a|\lambda}^V\}_a$ and gives result a according to the distribution $p(a|\psi_x^V, \lambda) = \text{tr}[M_{a|\lambda}^V \psi_x^V]$. Bob, in turn, upon receiving λ prepares the state ρ_λ^B which he then sends to the verifier as his teleported state.

Given the structure of this classical teleportation channel, we can now design a method to certify the quantumness of a

teleportation experiment that makes use of the full observed data:

$$\begin{aligned}\text{given} \quad & \{\sigma_{a|\psi_x}^B\}_{a,x}, \{\psi_x^V\}_x \\ \text{find} \quad & \{M_a^{VB}\}_a \\ \text{s.t.} \quad & \sigma_{a|\psi_x}^B = \text{tr}_V[M_a^{VB}(\psi_x^V \otimes \mathbb{1}^B)] \quad \forall a, x, \\ & M_a^{VB} \in \mathcal{S} \quad \forall a,\end{aligned}\quad (8)$$

where \mathcal{S} denotes the set of separable operators, *i.e.* operators of the form $\sum_\lambda \tau_\lambda \otimes \chi_\lambda$, with $\tau_\lambda \geq 0$ and $\chi_\lambda \geq 0$ for all λ .

Note that although the set of separable operators has a complicated structure [12], we can nevertheless relax \mathcal{S} in (8) to be the set of operators with positive partial transposition (PPT) [13], which has a simple characterisation in terms of a single semidefinite constraint. In this case the above test becomes an instance of a feasibility semidefinite program [14], which can be easily solved with available software [15]. Moreover, in the case of qubit teleportation, since the PPT criterion is necessary and sufficient for testing separability [16], Eq. (8) (without relaxation) is already an SDP, and therefore straightforward to solve. In higher dimensions other semidefinite relaxations of the set of separable operators have also been proposed and can be readily implemented [17].

As mentioned before, in the case that $\{\psi_x^V\}_x$ form a tomographically complete set the verifier can reconstruct the operators M_a^{VB} . Thus, in this case he can use any entanglement criterion [12] to check the separability of the operators M_a^{VB} . Moreover, as we will show now, if at least one of the measurement operators of Alice's measurement corresponds to the projection onto a maximally entangled state, any entangled state ρ^{AB} suffices to demonstrate that the teleportation process is genuinely quantum.

Let us first consider that Alice applies a full Bell state measurement of two qudits, composed by measurement operators $M_a^{VA} = (U_a \otimes \mathbb{1})|\Phi^+\rangle\langle\Phi^+|(U_a \otimes \mathbb{1})^\dagger$, where $|\Phi^+\rangle = \sum_i |i\rangle^V |i\rangle^A / \sqrt{d}$ is a maximally entangled state and U_a are appropriate unitary operations that generate the whole basis of Bell states. In this case

$$\begin{aligned}M_a^{VB} &= \text{tr}_A[(M_a^{VA} \otimes \mathbb{1}^B) \cdot (\mathbb{1}^V \otimes \rho^{AB})] \\ &= \text{tr}_A \left[\left((U_a^V \otimes \mathbb{1}^A) |\Phi^+\rangle\langle\Phi^+| (U_a^V \otimes \mathbb{1}^A)^\dagger \otimes \mathbb{1}^B \right) \cdot (\mathbb{1}^V \otimes \rho^{AB}) \right] \\ &= \frac{1}{d} \zeta^{VB},\end{aligned}\quad (9)$$

where $\zeta^{VB} = [(U_a^V \otimes \mathbb{1}^B) \rho^{VB} (U_a^V \otimes \mathbb{1}^B)^\dagger]^{T_B}$, and $\rho^{VB} = \rho^{AB}$. Since the partial transposition of an entangled state is either non-positive, or an entangled operator, if ρ^{AB} is entangled then we get a contradiction with (5). Thus, any entangled state ρ^{AB} can be used to implement a teleportation channel with no classical explanation.

A subject of debate in the literature was the fact that some demonstrations of teleportation were probabilistic [8], due to

the impossibility of performing a complete Bell state measurement with linear optics [18] or the use of inefficient detectors. We can roughly model these non-perfect measurements with the measurement operators $M_0^{\text{VA}} = |\Phi^+\rangle\langle\Phi^+|$ and $M_1^{\text{VA}} = \mathbb{1} - |\Phi^+\rangle\langle\Phi^+|$, where outcome 1 corresponds to the “bad” outcome. In this case, the effective channel applied to the input states is given by the channel operators $M_0^{\text{VB}} = \frac{1}{d}(\rho^{\text{VB}})^{T_B}$ and $M_1^{\text{VB}} = \mathbb{1}^V \otimes \rho^B - \frac{1}{d}(\rho^{\text{VB}})^{T_B}$. The fact that M_0^{VB} is entangled or has a negative eigenvalue whenever ρ^{AB} is entangled, implies that the verifier can also certify teleportation from every entangled state using a partial Bell state measurement. Notice that although a single outcome of Alice’s measurement suffices to demonstrate the quantumness of teleportation, no post-selection is required in this certification. This model for the partial Bell state measurement also encompasses the case where the Bell state measurement has a limited efficiency [5?].

In addition to the above results, method (8) also brings new ingredients to the task of certifying and quantifying the quantumness of a teleportation channel. In what follows we describe the idea of a *non-classical teleportation witness*, which, similarly to the idea of an entanglement witness [12], is an experimentally-friendly test that can be used to certify that the teleportation channel has no classical description. The optimisation problem (8) has the following dual formulation [14]:

$$\begin{aligned} &\text{given } \{\sigma_{a|\psi_x}^B\}_{a,x}, \{\psi_x^V\}_x \\ &\min_{F_{a|\psi_x}} \quad \text{tr} \sum_{a,x} F_{a|\psi_x}^B \sigma_{a|\psi_x}^B \\ &\text{s.t.} \quad \text{tr} \sum_x [(\psi_x^V \otimes F_{a|\psi_x}^B) \rho_{\text{sep}}^{\text{VB}}] \geq 0 \quad \forall a, \forall \rho_{\text{sep}}^{\text{VB}} \in \mathcal{S}. \end{aligned} \quad (10)$$

If the minimum value this optimisation problem achieves is non-negative then the data does not demonstrate teleportation. Conversely, a negative solution certifies that a non-classical teleportation process has taken place.

Moreover, from (10) we can conclude that for any sets $\{\psi_x^V\}_x$ and $\{F_{a|\psi_x}^B\}_{a,x}$ satisfying $\text{tr} \sum_x [(\psi_x^V \otimes F_{a|\psi_x}^B) \rho_{\text{sep}}^{\text{VB}}] \geq 0$ for all separable states ρ_{sep} (i.e. $W_a = \sum_x \psi_x^V \otimes F_{a|\psi_x}^B$ is an entanglement witness for all a), a negative value for $\text{tr} \sum_{a,x} F_{a|\psi_x}^B \sigma_{a|\psi_x}^B$ certifies non-classical teleportation. Hence, the operators $\{F_{a|\psi_x}^B\}_{a,x}$ serve as a teleportation witness for the set of input states $\{\psi_x^V\}_x$.

The present ideas also lead to a natural way of quantifying the quantumness of a teleportation process in terms of how much noise we have to add to the outcome states of a teleportation experiment such that it has a classical explanation. This is captured by the following optimisation problem:

$$\begin{aligned} &\text{given } \{\sigma_{a|\psi_x}^B\}_{a,x}, \{\psi_x^V\}_x, \mathcal{N} \\ &\text{minimise } r \\ &\text{s.t.} \quad \frac{\sigma_{a|\psi_x}^B + r\gamma_{a|\psi_x}^B}{1+r} = \text{tr}_V[M_a^{\text{VB}}(\psi_x^V \otimes \mathbb{1}^B)] \quad \forall a, x, \\ &\quad M_a^{\text{VB}} \in \mathcal{S} \quad \forall a, \\ &\quad \gamma_{a|\psi_x}^B \in \mathcal{N} \quad \forall a, x. \end{aligned} \quad (11)$$

		x					
$F_{a \psi_x}^B$		0	1	2	3	4	5
a	0	$\frac{\mathbb{1}}{3} - X$	$\frac{\mathbb{1}}{3} + X$	$\frac{\mathbb{1}}{3} - Y$	$\frac{\mathbb{1}}{3} + Y$	$\frac{\mathbb{1}}{3} - Z$	$\frac{\mathbb{1}}{3} + Z$
	1	$\frac{\mathbb{1}}{3} - X$	$\frac{\mathbb{1}}{3} + X$	$\frac{\mathbb{1}}{3} + Y$	$\frac{\mathbb{1}}{3} - Y$	$\frac{\mathbb{1}}{3} + Z$	$\frac{\mathbb{1}}{3} - Z$
	2	$\frac{\mathbb{1}}{3} + X$	$\frac{\mathbb{1}}{3} - X$	$\frac{\mathbb{1}}{3} + Y$	$\frac{\mathbb{1}}{3} - Y$	$\frac{\mathbb{1}}{3} - Z$	$\frac{\mathbb{1}}{3} + Z$
	3	$\frac{\mathbb{1}}{3} + X$	$\frac{\mathbb{1}}{3} - X$	$\frac{\mathbb{1}}{3} - Y$	$\frac{\mathbb{1}}{3} + Y$	$\frac{\mathbb{1}}{3} + Z$	$\frac{\mathbb{1}}{3} - Z$

Table I. Teleportation witness for the two-qubit Werner state (12). The verifier provides the states $\{\psi_x\}_x = \{(|0\rangle \pm |1\rangle)/\sqrt{2}, (|0\rangle \pm i|1\rangle)/\sqrt{2}, |0\rangle, |1\rangle\}$ to Alice. By measuring the observables $F_{a|\psi_x}^B$ when Bob forwards the state $\sigma_{a|\psi_x}^B$ to the verifier, the value $\text{tr} \sum_{a,x} F_{a|\psi_x}^B \sigma_{a|\psi_x}^B = 6(\frac{1}{3} - p)$ is obtained, which is negative for all $p > 1/3$. Thus all entangled two-qubit Werner states are witnessed as useful for teleportation.

The set \mathcal{N} corresponds to the set of states $\gamma_{a|\psi_x}$ that are considered as noise. Each choice of \mathcal{N} defines a different robustness quantifier. Natural examples include the set of all possible teleported states (generalised robustness), or the maximally mixed state (random robustness). If the solution of this problem is strictly positive, this means that a finite amount of noise needs to be added to make the teleportation process classical.

Before we finish this letter let us provide some explicit examples. Consider teleportation of the states $\{\psi_x\}_x = \{(|0\rangle \pm |1\rangle)/\sqrt{2}, (|0\rangle \pm i|1\rangle)/\sqrt{2}, |0\rangle, |1\rangle\}$ using the two-qubit Werner state

$$\rho^{\text{AB}} = p|\Phi^+\rangle\langle\Phi^+| + (1-p)\frac{\mathbb{1}^{\text{AB}}}{4} \quad (12)$$

and full Bell state measurement, $\{M_a^{\text{VA}}\}_a = \{|\Phi^+\rangle\langle\Phi^+|, |\Psi^+\rangle\langle\Psi^+|, |\Phi^-\rangle\langle\Phi^-|, |\Psi^-\rangle\langle\Psi^-|\}$. The teleportation witness is given in Table I. We have $\text{tr} \sum_{a,x} F_{a|\psi_x}^B \sigma_{a|\psi_x}^B = 2(1 - 3p)$, and therefore teleportation is certified for all $p > 1/3$, which coincides with the separability bound of the state (12). Finally, we also calculated the random robustness and generalised robustness of teleportation for the above teleportation scheme. We find that the random robustness is given by $(3p - 1)$, while the generalised robustness is $(3p - 1)/4$.

Consider now the so-called ‘tiles’ bound entangled state [23]:

$$\rho_{\text{tiles}} = \frac{1}{4} \left(\mathbb{1} - \sum_{i=0}^4 |\phi_i\rangle\langle\phi_i| \right), \quad (13)$$

where the states $|\phi_i\rangle$ form an unextendible product basis (UPB):

$$\begin{aligned} |\phi_0\rangle &= \frac{1}{\sqrt{2}} |0\rangle (|0\rangle - |1\rangle), \quad |\phi_1\rangle = \frac{1}{\sqrt{2}} |2\rangle (|1\rangle - |2\rangle), \\ |\phi_2\rangle &= \frac{1}{\sqrt{2}} (|0\rangle - |1\rangle) |2\rangle, \quad |\phi_3\rangle = \frac{1}{\sqrt{2}} (|1\rangle - |2\rangle) |0\rangle, \end{aligned} \quad (14)$$

		x					
a	$F_{a \psi_x}^B$	0	1	2	3	4	5
	0	ψ_2	ψ_3	ψ_1	ψ_0	ψ_4	$-3\epsilon\mathbb{I}$
	1	0	0	0	0	0	0

Table II. Non-classical teleportation witness for the two-qutrit bound entangled ‘tiles’ state (13). The verifier provides the states $\{\psi_x\}_x = \{|0\rangle\langle 0|, |2\rangle\langle 2|, (|0\rangle - |1\rangle)(\langle 0| - \langle 1|)/2, (|1\rangle - |2\rangle)(\langle 1| - \langle 2|)/2, (|0\rangle + |1\rangle + |2\rangle)(\langle 0| + \langle 1| + \langle 2|)/3, \mathbb{I}/3\}$ to Alice. Here, $\epsilon \in (0, 0.02842]$. By measuring the observables $F_{a|\psi_x}^B$ when Bob forwards the state $\sigma_{a|\psi_x}^B$ to the verifier, the value $\text{tr} \sum_{a,x} F_{a|\psi_x}^B \sigma_{a|\psi_x}^B = -\epsilon/3$ is obtained.

$$|\phi_4\rangle = \frac{1}{3}(|0\rangle + |1\rangle + |2\rangle)(|0\rangle + |1\rangle + |2\rangle).$$

According to the benchmark based on the average fidelity (2) this state is useless for teleportation [10]. For the set of input states $\{\psi_x\}_x = \{|0\rangle, |2\rangle, (|0\rangle - |1\rangle)/\sqrt{2}, (|1\rangle - |2\rangle)/\sqrt{2}, (|0\rangle + |1\rangle + |2\rangle)/\sqrt{3}, \mathbb{I}/3\}$ and partial Bell state measurement $\{M_a^{VA}\}_a = \{|\Phi^+\rangle\langle\Phi^+|, \mathbb{I} - |\Phi^+\rangle\langle\Phi^+|\}$ we generate the non-classical teleportation witness given in Table II. The state achieves the value $\text{tr} \sum_{a,x} F_{a|\psi_x}^B \sigma_{a|\psi_x}^B = -\epsilon/3$. This clearly illustrates the fact that the teleportation process using bound entangled states can not be classically simulated.

A couple of additional comments are in order. First, note that the set of input states $\{\psi_x\}_x$ in this instance is not even tomographically complete, and yet teleportation can nevertheless be certified. Second, here we considered only a partial Bell state measurement. Since M_1^{VA} is a separable operator in this instance, it is for this reason that $F_{1|\psi_x}^B$ vanish. Finally, we note that $W_0 = \sum_x \psi_x^V \otimes F_{1|\psi_x}^B = \sum_i |\phi_i\rangle\langle\phi_i| - \epsilon\mathbb{I}$ is precisely the entanglement witness which is violated by the ‘tiles’ UPB state (13) for $\epsilon \in (0, 0.02842]$ [24]. This demonstrates that the constraint in (10) is indeed satisfied.

We finish this paper by commenting on the fact that the present study also makes clear some connections between quantum teleportation and other ideas discussed in quantum foundations, such as EPR steering [19] and Bell inequalities with quantum inputs [20]. EPR steering is sometimes phrased in terms of a task where Bob wants to certify that he shares entanglement with Alice, but he does not trust her. He then asks her to perform some measurements on her share of the state and applies a test based on the post-measured states he obtains. Notice that this is exactly the teleportation scenario as presented in Fig. 1, but with the crucial distinction that the inputs to Alice’s measuring devices are classical variables x , as opposed to the quantum variables ψ_x in teleportation. Crucially, due to this difference, not every entangled state is useful for demonstrating steering [22]. Another similar situation is the recently introduced Bell inequalities with quantum inputs [20], which was later interpreted as the task of measurement-device-independent entanglement detection [21]. The scenario is the same as in quantum teleportation, but now Bob also applies a measurement with a quantum input to his share

of the state. In this case all entangled states demonstrate non-classical correlations. Under the view of our results this now becomes clear: if Alice and Bob apply a Bell state measurement and receive a tomographically complete set of measurements, the verifier can reconstruct the state they share, and thus determine whether it is entangled or not.

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* daniel.cavalcanti@icfo.es

- [1] C. H. Bennett, G. Brassard, C. Crépeau, R. Jozsa, A. Peres, and W. K. Wootters, *Teleporting an unknown quantum state via dual classical and Einstein-Podolsky-Rosen channels*, Phys. Rev. Lett. **70**, 1895 (1993).
- [2] M. Zukowski, A. Zeilinger, M. A. Horne, and A. K. Ekert, “Event-ready-detectors” Bell experiment via entanglement swapping, Phys. Rev. Lett. **71**, 4287 (1993).
- [3] H.-J. Briegel, W. Dür, J. I. Cirac, and P. Zoller, *Quantum repeaters: The role of imperfect local operations in quantum communication*, Phys. Rev. Lett. **81**, 5932–5935 (1998).
- [4] D. Gottesman, I. L. Chuang, *Demonstrating the viability of universal quantum computation using teleportation and single-qubit operations*, Nature **402**, 390–393 (1999).
- [5] D. Boschi, S. Branca, F. De Martini, L. Hardy, and S. Popescu, *Experimental Realization of Teleporting an Unknown Pure Quantum State via Dual Classical and Einstein-Podolsky-Rosen Channels* Phys. Rev. Lett. **80**, 1121 (1998).
- [6] D. Bouwmeester, J.-W. Pan, K. Mattle, M. Eibl, H. Weinfurter, and A. Zeilinger, *Experimental quantum teleportation*, Nature **390**, 575–579 (1997). Phys. Rev. Lett. **80**, 3891 (1998).
- [7] A. Furusawa, et al. *Unconditional quantum teleportation*, Science **282**, 706–709 (1998).
- [8] S. Pirandola, J. Eisert, C. Weedbrook, A. Furusawa, and S. L. Braunstein, *Advances in quantum teleportation*, Nature Photonics **9**, 641–652 (2015).
- [9] M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information* (Cambridge University Press, 2000).
- [10] M. Horodecki, P. Horodecki, and R. Horodecki, *General teleportation channel, singlet fraction, and quasidistillation*, Phys. Rev. A **60**, 1888 (1999).
- [11] Ll. Masanes, *All Bipartite Entangled States Are Useful for Information Processing*, Phys. Rev. Lett. **96**, 150501 (2006).
- [12] R. Horodecki, P. Horodecki, M. Horodecki, and K. Horodecki, *Quantum entanglement*, Rev. Mod. Phys. **81**, 865 (2009).
- [13] A. Peres, *Separability Criterion for Density Matrices*, Phys. Rev. Lett. **77**, 1413 (1996).
- [14] S. Boyd and L. Vandenberghe, *Convex Optimization*, (Cambridge University Press, 2004).

- [15] M. Grant, S. Boyd, *CVX: Matlab Software for Disciplined Convex Programming*, version 2.1, <http://cvxr.com/cvx> (2014).
- [16] M. Horodecki, P. Horodecki, R. Horodecki, *Separability of Mixed States: Necessary and Sufficient Conditions*, Physics Letters A **223**, 1 (1996).
- [17] A. C. Doherty, Pablo A. Parrilo, and F. M. Spedalieri, *Distinguishing Separable and Entangled States*, Phys. Rev. Lett. **88**, 187904 (2002).
- [18] N. Lütkenhaus, J. Calsamiglia, and K.-A. Suominen, *Bell measurements for teleportation*, Phys. Rev. A **59**, 3295 (1999).
- [19] H. M. Wiseman, S. J. Jones, and A. C. Doherty, *Steering Entanglement, Nonlocality, and the Einstein-Podolsky-Rosen Paradox*, Phys. Rev. Lett. **98** 140402 (2007).
- [20] F. Buscemi, *All Entangled Quantum States Are Nonlocal*, Phys. Rev. Lett. **108**, 200401 (2012).
- [21] C. Branciard, D. Rosset, Y.-C. Liang, N. Gisin, *Measurement-Device-Independent Entanglement Witnesses for All Entangled Quantum States*, Phys. Rev. Lett. **110**, 060405 (2013).
- [22] D. Cavalcanti and P. Skrzypczyk, *Quantum steering: a short review with focus on semidefinite programming*, *arXiv:1604.00501*.
- [23] C. H. Bennett, D. P. DiVincenzo, T. Mor, P. W. Shor, J. A. Smolin, B. Terhal, *Unextendible product bases and bound entanglement*, Phys. Rev. Lett. **82**, 5385 (1999).
- [24] O. Gühne, P. Hyllus, D. Bruss, A. Ekert, M. Lewenstein, C. Macchiavello, A. Sanpera, *Detection of entanglement with few local measurements*, Phys. Rev. A **66**, 062305 (2002).